MAV Annual Conference 2024
General Maths exams: using the CAS calculator efficiently and effectively

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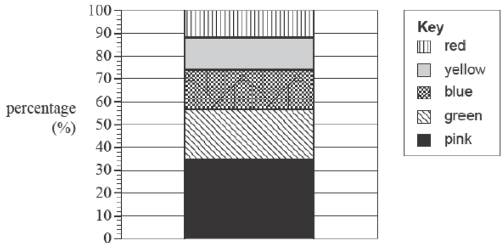
Data analysis:

MC Question 1

A group of students were asked to name their favourite colour.

The results are displayed in the percentage segmented bar chart below.



The percentage of students who named blue as their favourite colour is closest to

No CAS benefit

Question 2

Freddie organised a function at work. He surveyed the staff about their preferences.

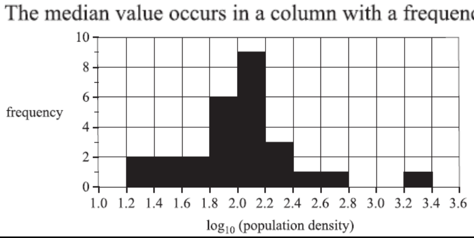
He asked them about their *payment preference* (cash or electronic payment) and their *budget preference* (less than $50 or more than $50).

The variables in this survey, *payment preference* and *budget preference*, are

No CAS benefit

Question 3

The median value occurs in a column with a frequency of



No CAS benefit

Question 4

There is one outlier at the upper end of the histogram.
This value could be

solve$(\log_{10}(p)=3.2, p)$

{p=1584.893192}

solve$(\log_{10}(p)=3.42, p)$

{p=2630.267992}

Question 5

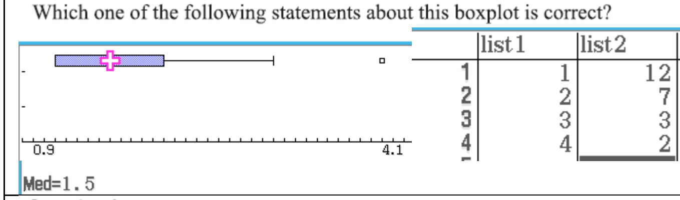
The number of siblings of each member of a class of 24 students was recorded.

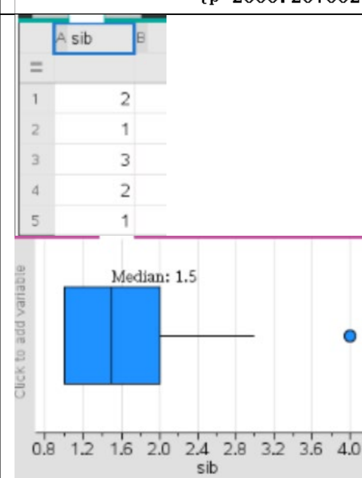
The results are displayed in the table below.

2	1	3	2	1	1	1	4	1	1	1	1
1	2	1	2	2	1	3	4	2	2	3	1

A boxplot was constructed to display the spread of the data.

Which one of the following statements about this boxplot is correct?





Question 6

More than 11 000 athletes from more than 200 countries competed in the Tokyo Summer Olympic Games.

An analysis of the number of athletes per country produced the following five-number summary.

Minimum	First quartile	Median	Third quartile	Maximum
2	5	11	48	613

Calculation of lower fence needed. Scientific would be enough.

48+1.5×(48−5)

112.5

Question 7

Fiona plays nine holes of golf each week, and records her *score*.

Her mean *score* for all rounds in 2024 is 55.7

In one round, when she recorded a *score* of 48, her standardised score was $z = -1.75$

The standard deviation for *score* in 2024 is

Standardised z-rule – CAS solving.

$$\text{solve}\left(\frac{48-55.7}{s}=-1.75, s\right)$$

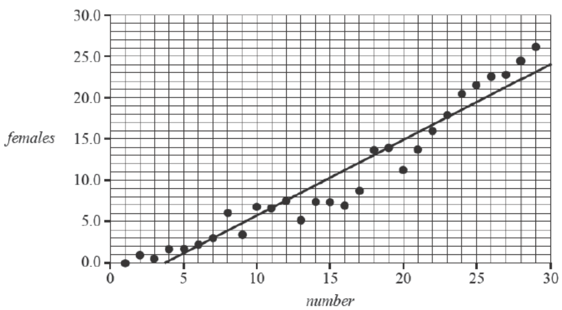
$\{s=4.4\}$

$$\text{solve}\left(\frac{48-55.7}{s}=-1.75, s\right) \quad s=4.4$$

Question 8

The scatterplot below displays the average number of female athletes per competing nation, *females*, against the number of the Summer Olympic Games, *number*, from the first Olympic Games, in 1896, to the 29th Olympic Games, held in 2021.

A least squares line has been fitted to the scatterplot.



list1	list2
5	1
30	24

Linear Reg

y=a+b•x

a	=−3.6
b	=0.92
r	=1
r^2	=1

A xx	B yy
5	1
30	24
	=LinRegB
Title	Linear R...
RegEqn	a+b*x
a	-3.6
b	0.92
r^2	1.

The least squares equation for the relationship between the average number of male athletes per competing nation, *males*, and the number of the Summer Olympic Games, *number*, is

$$males = 67.5 - 1.27 \times number$$

Question 9

The summary statistics for the variables *number* and *males* are shown in the table below.

	number	***males***
mean	15.0	48.4
standard deviation	8.51	19.0

The value of Pearson's correlation coefficient, *r*, rounded to three decimal places, is closest to

Linear regression rule - solving

$$\text{solve}\left(-1.27=r\cdot\frac{19}{8.51}, r\right)$$

$\{r=-0.5688263158\}$

$$\text{solve}\left(-1.27=r\cdot\frac{19}{8.51}, r\right)$$

$r=-0.568826315789$

Question 10

At which Summer Olympic Games will the predicted average number of *males* be closest to 25.6?

Substitution and solving

$\text{solve}(25.6=67.5-1.27\times n, n)$

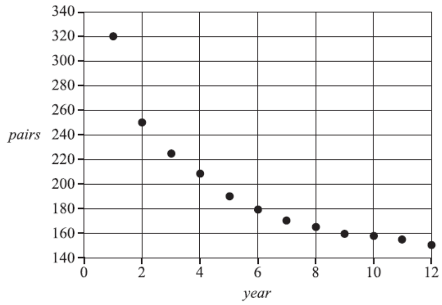
$\{n=32.99212598\}$

$\text{solve}(25.6=67.5-1.27\cdot n, n)$

$n=32.9921259843$

The number of breeding pairs of a small parrot species has been declining over recent years.

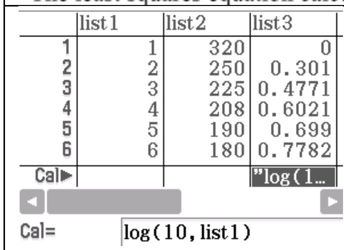
year	*pairs*
1	320
2	250
3	225
4	208
5	190
6	180
7	170
8	165
9	160
10	158
11	155
12	150

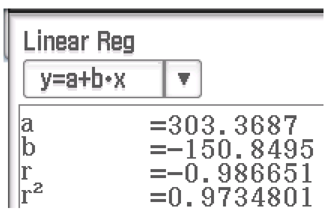


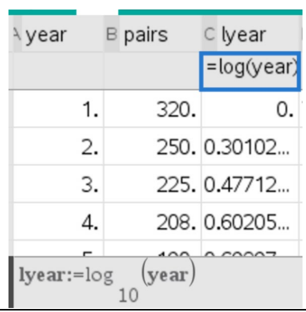
Question 11

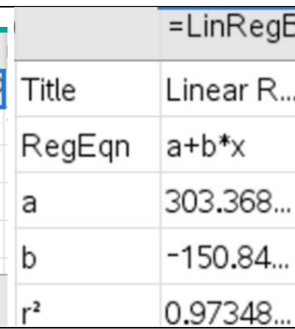
The scatterplot can be linearised using a logarithmic (base 10) transformation applied to the explanatory variable.

The least squares equation calculated from the transformed data is closest to





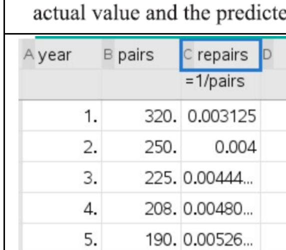


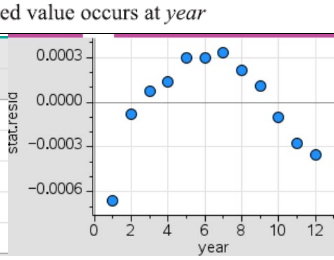


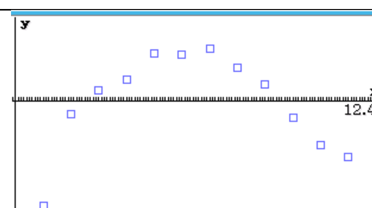
Question 12

A reciprocal transformation applied to the variable *pairs* can also be used to linearise the scatterplot.

When a least squares line is fitted to the plot of $\frac{1}{pairs}$ versus *year*, the largest difference between the actual value and the predicted value occurs at *year*







The time series plot below shows the number of new staff, *new*, for each year, *year*, from 2011 to 2022 (inclusive).

Question 13

The time series is smoothed using seven-median smoothing.

The smoothed value of *new* for the *year* 2016 is

No CAS benefit.

Question 14

The number of new staff in 2023 is added to the total number of new staff from the previous 12 years.

For these 13 years, the mean number of new staff is 11.

The number of new staff in 2023 is

$\text{solve}\left(\frac{14+11+7+6+11+13+12+6+10+15+12+10+n}{13}=11, n\right)$

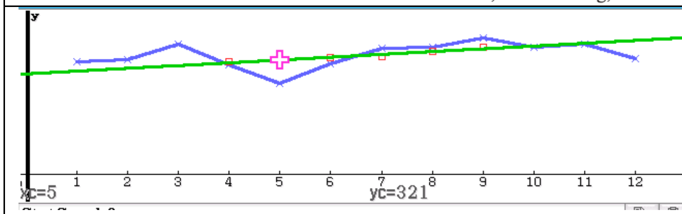
$\{n=16\}$

Question 15

The table below shows the total number of cans of soft drink sold each month at a suburban cafe in 2023.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Cans sold	316	321	365	306	254	308	354	357	381	355	365	324

The six-mean smoothed value of the number of cans sold, with centring, for month 5 is closest to



$\frac{321+365+306+254+308+354}{6}$ 318.

$\frac{365+306+254+308+354+357}{6}$ 324.

$\frac{318+324}{2}$ 321.

Question 16

The table below shows the seasonal indices for the monthly takings of a bistro.

The seasonal indices for months 3 and 6 are missing.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Seasonal index	1.08	1.13		0.92	0.67		1.09	1.35	0.82	0.88	1.01	0.98

The seasonal index for month 3 is twice the seasonal index for month 6.

The seasonal index for month 3 is closest to

$\text{solve}\left(1.08+1.13+s+0.92+0.67+\frac{s}{2}+1.09+1.35+0.82+0.88+1.01+0.98=12, s\right)$

$\{s=1.38\}$

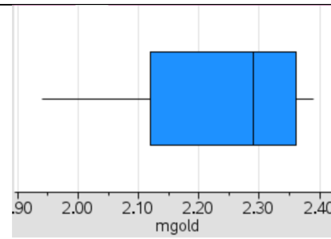
Data analysis

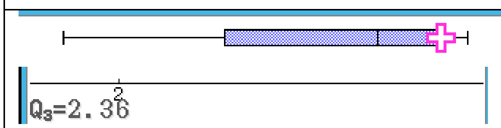
Question 1

Table 1

year	*Mgold* (m)
1928	1.94
1932	1.97
1936	2.03
1948	1.98
1952	2.04
1956	2.12
1960	2.16
1964	2.18
1968	2.24
1972	2.23
1976	2.25
1980	2.36
1984	2.35
1988	2.38
1992	2.34
1996	2.39
2000	2.35
2004	2.36
2008	2.36
2012	2.33
2016	2.38
2020	2.37

c. Construct a boxplot for the *Mgold* data in Table 1 on the grid below.





d. A least squares line can also be used to model the association between *Mgold* and *year*.

Using the data from Table 1, determine the equation of the least squares line for this data set.

Use the template below to write your answer.

Round the values of the intercept and slope to three significant figures.

Linear Reg

y=a+b·x

a	=−7.97105
b	=5.1613E−3
r	=0.9259773
r²	=0.8574339

sround(−7.97105, 3)
−7.97

sround(5.1613E−3, 3)
0.00516

C	D
	=LinRegBx
Title	Linear R...
RegEqn	a+b*x
a	-7.9710...
b	0.00516...
r²	0.85743...
r	0.92597...

Question 3 (10 marks)

When a least squares line is fitted to the scatterplot, the equation is found to be:

$$Wbest = 0.300 + 0.860 \times Wgold$$

The correlation coefficient is 0.9318

b. Draw the least squares line on the scatterplot on **page 6**.

Wbest=0.300+0.860×1.90

Wbest=1.934

Wbest=0.300+0.860×2.06

Wbest=2.0716

c. Determine the value of the coefficient of determination as a percentage. Round your answer to one decimal place.

$(0.9318)^2 \cdot 100$ 86.825124

Recursion and Financial modelling

Question 17

A first-order linear recurrence relation of the form

$$u_0 = a, \qquad u_{n+1} = Ru_n + d$$

generates the terms of a sequence. A geometric sequence will be generated if

No CAS benefit

Question 18

Trevor took out a reducing balance loan of $400 000, with interest calculated weekly.

The balance of the loan, in dollars, after n weeks, T_n, can be modelled by the recurrence relation

$$T_0 = 400\,000, \qquad T_{n+1} = 1.00075T_n - 677.55$$

Assume that there are exactly 52 weeks in a year.

The interest rate, per annum, for this loan is

$\text{solve}\left(1+\frac{\frac{r}{52}}{100}=1.00075, r\right)$

{r=3.9}

Question 19

Liv bought a new car for $35 000. The value of the car will be depreciated by 18% per annum using the reducing balance method.

A recurrence relation that models the year-to-year value of her car is of the form

$$L_0 = 35\,000, \qquad L_{n+1} = k \times L_n$$

The value of k is

No CAS benefit (scientific would suffice)

Question 20

Dainika invested $2000 for three years at 4.4% per annum, compounding quarterly.

To earn the same amount of interest in three years in a simple interest account, the annual simple interest rate would need to be closest to

$\text{solve}\left(2000\left(1+\frac{\frac{4.4}{4}}{100}\right)^{12}-2000=\frac{2000\times r\times 3}{100}, r\right)$

{r=4.67620655}

Question 21

Lee took out a loan of $121 000, with interest compounding monthly. He makes monthly repayments of $2228.40 for five years until the loan is repaid in full.

The total interest paid by Lee is closest to

2228.4· 12· 5−121000 12704.

Amortization

PM1	1
PM2	60
I%	4.000015224
PV	121000
PMT	−2228.4
P/Y	12
C/Y	12
BAL	0.00000001267
INT	
PRN	
ΣINT	−12704
ΣPRN	−121000

Question 21

If Audrey uses reducing balance depreciation, the depreciation rate, per annum is closest to

Compound Interest

N	4
I%	−33.1259695
PV	3000
PMT	0
FV	−600
P/Y	1
C/Y	1

$\text{solve}\left(3000\cdot\left(1-\frac{r}{100}\right)^4=600, r\right)$

$r=33.1259695024$ or $r=166.874030498$

Stewart takes out a reducing balance loan of $240 000, with interest calculated monthly.

Stewart makes regular monthly repayments.

Three lines of the amortisation table are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	240 000.00
1	2741.05	960.00	1781.05	238 218.95
2	2741.05			

Question 22

The principal reduction associated with Payment number 2 is closest to

Lots of small steps.

1\.

$i=\frac{960}{240000}*100$

i=0.4

2\.

$238218.95\times\frac{0.4}{100}$

952.8758

3\.

2741.05−952.8758

1788.1742

Finance Solver

N:	2.
I(%):	0.4
PV:	240000.
Pmt:	-2741.05
FV:	-236430.7758
PpY:	1

238218.95−236430.7758 1788.1742

Question 23

The number of years that it will take Stewart to repay the loan in full is closest to

Compound Interest

N	107.9999207
I%	0.4
PV	240000
PMT	−2741.05
FV	0
P/Y	1
C/Y	1

$\frac{107.9999207}{12}$

8.999993392

Finance Solver

N:	107.99992072507
I(%):	0.4
PV:	240000.
Pmt:	-2741.05
FV:	0.
PpY:	1

$\frac{107.99992072507}{12}$ 8.99999339376

Question 24

André invested \$18 000 in an account for five years, with interest compounding monthly.

He adds an extra payment into the account each month immediately after the interest is calculated.

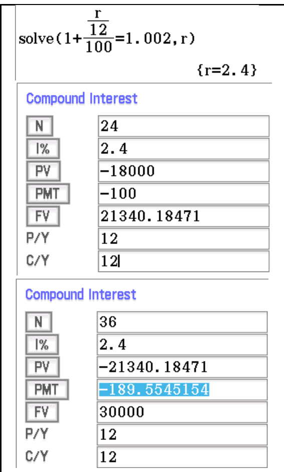
For the first two years, the balance of the account, in dollars, after n months, A_n, can be modelled by the recurrence relation

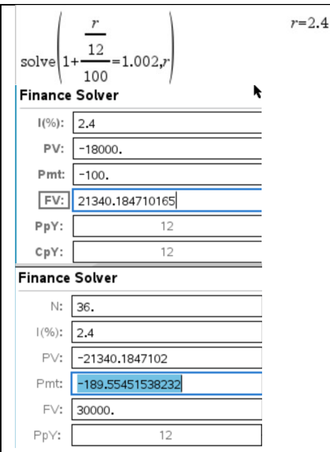
$$A_0 = 18000, \qquad A_{n+1} = 1.002A_n + 100$$

After two years, André decides he would like the account to reach a balance of \$30 000 at the end of the five years.

He must increase the value of the monthly extra payment to achieve this.

The minimum value of the new payment for the last three years is closest to





Recursion and Financial modelling

Question 5 (4 marks)

Emi operates a mobile dog-grooming business.

The value of her grooming equipment will depreciate.

Based on average usage, a rule for the value, in dollars, of the equipment, V_n, after n weeks is

$$V_n = 15\,000 - 60n$$

Assume that there are exactly 52 weeks in a year.

b. Emi plans to replace the grooming equipment after four years.

What will be its value, in dollars, at this time?

V4=15000−60×4×52

V4=2520

d. The value of the grooming equipment decreases from one year to the next by the same percentage of the original $15 000 value.

What is this annual flat rate percentage?

$$\frac{60\cdot 52}{15000}\cdot 100 \qquad 20.8$$

Question 6 (2 marks)

Emi invested profits of $10 000 into a savings account that earns interest compounding fortnightly, for one year.

The effective interest rate, rounded to two decimal places, is 5.07%.

Assume that there are exactly 26 fortnights in a year.

a. What is the nominal percentage rate of interest for the account?

Round your answer to two decimal places.

convNom(26, 5.07)

4.950367617

nom(5.07,26) 4.95036761696

Question 7 (4 marks)

Emi decides to invest a $300 000 inheritance into an annuity.

Let E_n be the balance of Emi's annuity after n months.

A recurrence relation that can model the value of this balance from month to month is

$$E_0 = 300\,000, \qquad E_{n+1} = 1.003E_n - 2159.41$$

b. For how many years will Emi receive the regular payment?

$$\text{solve}\left(1+\frac{\frac{r}{12}}{100}=1.003, r\right) \qquad \{r=3.6\}$$

Compound Interest

N	180.0000209
I%	3.6
PV	−300000
PMT	2159.41
FV	0
P/Y	12
C/Y	12

$$\frac{180.0000209}{12} \qquad 15.00000174$$

$$\text{solve}\left(1+\frac{\frac{r}{12}}{100}=1.003, r\right) \qquad r=3.6$$

Finance Solver

N:	180.00002087874
I(%):	3.6
PV:	-300000.
Pmt:	2159.41
FV:	0.
PpY:	12

$$\frac{180.00002087874}{12} \qquad 15.0000017399$$

c. Calculate the annual compound interest rate for this annuity.

$$\text{solve}\left(1+\frac{\frac{r}{12}}{100}=1.003, r\right) \qquad \{r=3.6\}$$

$$\text{solve}\left(1+\frac{\frac{r}{12}}{100}=1.003, r\right) \qquad r=3.6$$

d. If Emi wanted the annuity to act as a perpetuity, what monthly payment, in dollars, would she receive?

Compound Interest

N	1
I%	3.6
PV	−300000
PMT	900
FV	300000
P/Y	12
C/Y	12

Finance Solver

N:	1.
I(%):	3.6
PV:	-300000.
Pmt:	900.
FV:	300000.
PpY:	12

Question 8 (2 marks)

Emi takes out a reducing balance loan of $500 000.

The interest rate is 5.3% per annum, compounding monthly.

Emi makes regular monthly repayments of $3071.63 for the duration of the loan, with only the final repayment amount being slightly different from all the other repayments.

Determine the total cost of Emi's loan, rounding your answer to the nearest cent, and state the number of payments required to fully repay the loan.

Compound Interest

N	288.001363
I%	5.3
PV	500000
PMT	−3071.63
FV	0
P/Y	12
C/Y	12

Compound Interest

N	288
I%	5.3
PV	500000
PMT	−3071.63
FV	−4.17734464
P/Y	12
C/Y	12

288×3071.63+4.17734464

884633.6173

Finance Solver

N:	288.00136298168
I(%):	5.3
PV:	500000.
Pmt:	-3071.63
FV:	0.
PpY:	12

Finance Solver

N:	288.
I(%):	5.3
PV:	500000.
Pmt:	-3071.63
FV:	-4.177344641
PpY:	12

288· 3071.63+4.177344641

884633.617345

Matrices

Question 25

Matrix J is a 2×3 matrix.

Matrix K is a 3×1 matrix.

Matrix L is added to the product JK.

The order of matrix L is

No CAS benefit.

Question 26

A market stall sells three types of candles.

The cost of each type of candle is shown in matrix C below.

$$C = \begin{bmatrix} 25 & 32 & 43 \end{bmatrix}$$

Towards the end of the day, the cost of each item is discounted by 15%.

Which one of the following expressions can be used to determine each discounted price?

No CAS benefit.

Question 27

Consider the following matrix, where $h \neq 0$.

$$\begin{bmatrix} 4 & g \\ 8 & h \end{bmatrix}$$

The inverse of this matrix does **not** exist when g is equal to

$\text{solve}\left(\det\left(\begin{bmatrix} 4 & g \\ 8 & h \end{bmatrix}\right)=0,g\right)$ $\quad g=0.5\cdot h$

$\text{solve}\left(\det\left(\begin{bmatrix} 4 & g \\ 8 & h \end{bmatrix}\right),g\right)$

$\{g=0.5\cdot h\}$

Question 28

A primary school is hosting a sports day.

Students represent one of four teams: blue (B), green (G), red (R) or yellow (Y).

Students compete in one of three sports: football (F), netball (N) or tennis (T).

Matrix W shows the number of students competing in each sport and the team they represent

$$\begin{array}{cc} & \begin{array}{cccc} B & G & R & Y \end{array} \\ W = & \begin{bmatrix} 85 & 60 & 64 & 71 \\ 62 & 74 & 80 & 64 \\ 63 & 76 & 66 & 75 \end{bmatrix} \begin{array}{c} F \\ N \\ T \end{array} \end{array}$$

Matrix W is multiplied by the matrix $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ to produce matrix X.

$$\begin{bmatrix} 85 & 60 & 64 & 71 \\ 62 & 74 & 80 & 64 \\ 63 & 76 & 66 & 75 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 280 \\ 280 \\ 280 \end{bmatrix}$$

$$\begin{bmatrix} 85 & 60 & 64 & 71 \\ 62 & 74 & 80 & 64 \\ 63 & 76 & 66 & 75 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 280. \\ 280. \\ 280. \end{bmatrix}$$

Question 29

This information can be represented by matrix G_1, shown below.

$$G_1 = \begin{bmatrix} Q & R & S & T & U \end{bmatrix}$$

Let G_n be the order of play in week n.

The playing order changes each week and can be determined by the rule $G_{n+1} = G_n \times P$

where $P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Which player has a bye in week 4?

$$[Q\ R\ S\ T\ U]\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad [S\ Q\ T\ U\ R]$$

$$\text{ans}\times\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad [U\ T\ R\ Q\ S]$$

Question 30

Data has been collected on the female population of a species of mammal located on a remote island.

The female population has been divided into three age groups, with the initial population (at the time of data collection), the birth rate, and the survival rate of each age group shown in the table below.

	Age group (years)		
	0–2	2–4	4–6
Initial population	2100	6400	4260
Birth rate	0	1.8	1.2
Survival rate	0.7	0.6	0

The Leslie matrix (L) that may be used to model this particular population is

No CAS benefit.

Question 31

A species of bird has a life span of three years.

The females in this species do not reproduce in their first year but produce an average of four female offspring in their second year, and three in their third year.

The Leslie matrix, L, below is used to model the female population distribution of this species of bird.

$$L=\begin{bmatrix}0 & 4 & 3\\ 0.2 & 0 & 0\\ 0 & 0.4 & 0\end{bmatrix}$$

The element in the second row, first column states that on average 20% of this population will

No CAS benefit.

Question 31

The matrix below shows the results of a round-robin chess tournament between five players: H, I, J, K and L. In each game, there is a winner and a loser.

Two games still need to be played.

$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{ccccc} H & I & J & K & L \end{array} \\ \textit{winner} \begin{array}{c} H \\ I \\ J \\ K \\ L \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & \ldots & 1 & \ldots \\ 1 & \ldots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & \ldots & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Which one of the following is **not** a potential outcome after the final two games have been played?

$$\begin{bmatrix}0&1&0&1&0\\0&0&1&1&1\\1&0&0&1&0\\0&0&0&0&1\\1&0&1&0&0\end{bmatrix}\Rightarrow a$$

$$(a+a^2)\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$

Question 32

The expected number of people at each location can be determined by the matrix recurrence rule

$S_{n+1}=TS_n+A$

where

$$T=\begin{array}{c} \textit{this day} \\ \begin{array}{cccc} G & C & R & M \end{array} \\ \begin{bmatrix}0.4 & 0.2 & 0.4 & 0\\ 0.4 & 0.1 & 0.3 & 0.3\\ 0.1 & 0.4 & 0.1 & 0.2\\ 0.1 & 0.3 & 0.2 & 0.5\end{bmatrix}\end{array}\begin{array}{c} G \\ C \\ R \\ M \end{array} \ \textit{next day}$$

and

$$A=\begin{bmatrix}300\\200\\100\\300\end{bmatrix}\begin{array}{c} G \\ C \\ R \\ M \end{array}$$

Given the state matrix $S_3=\begin{bmatrix}5620\\6386\\4892\\6902\end{bmatrix}\begin{array}{c} G \\ C \\ R \\ M \end{array}$

the number of people watching the event at the Botanical Gardens (G) from Thursday to Sunday has

$$\text{solve}\left(\begin{bmatrix}0.4&0.2&0.4&0\\0.4&0.1&0.3&0.3\\0.1&0.4&0.1&0.2\\0.1&0.3&0.2&0.5\end{bmatrix}\begin{bmatrix}g\\c\\r\\m\end{bmatrix}+\begin{bmatrix}300\\200\\100\\300\end{bmatrix}=\begin{bmatrix}5620\\6386\\4892\\6902\end{bmatrix},\{g,c,r,m\}\right)$$

{g=5496, c=6168, r=4720, m=6516}

$$\text{solve}\left(\begin{bmatrix}0.4&0.2&0.4&0\\0.4&0.1&0.3&0.3\\0.1&0.4&0.1&0.2\\0.1&0.3&0.2&0.5\end{bmatrix}\begin{bmatrix}g\\c\\r\\m\end{bmatrix}+\begin{bmatrix}300\\200\\100\\300\end{bmatrix}=\begin{bmatrix}5496\\6168\\4720\\6516\end{bmatrix},\{g,c,r,m\}\right)$$

{g=5832, c=6076, r=4120, m=5972}

$$\text{solve}\left(\begin{bmatrix}0.4&0.2&0.4&0\\0.4&0.1&0.3&0.3\\0.1&0.4&0.1&0.2\\0.1&0.3&0.2&0.5\end{bmatrix}\begin{bmatrix}g\\c\\r\\m\end{bmatrix}+\begin{bmatrix}300\\200\\100\\300\end{bmatrix}=\begin{bmatrix}5832\\6076\\4120\\5972\end{bmatrix},\{g,c,r,m\}\right)$$

{g=4924, c=4732, r=6540, m=4904}

5620−4924

696

Matrices

Question 9 (3 marks)

Vince works on a construction site.

The amount Vince gets paid depends on the type of shift he works, as shown in the table below.

Shift type	Normal	Overtime	Weekend
Hourly rate of pay ($ per hour)	36	54	72

This information is shown in matrix R below.

$$R = \begin{bmatrix} 36 & 54 & 72 \end{bmatrix}$$

a. Matrix R^T is the transpose of matrix R.

trn([36 54 72])

$$\begin{bmatrix} 36 \\ 54 \\ 72 \end{bmatrix}$$

$$\begin{bmatrix} 36 & 54 & 72 \end{bmatrix}^T \qquad \begin{bmatrix} 36. \\ 54. \\ 72. \end{bmatrix}$$

During one week, Vince works 28 hours at the normal rate of pay, 6 hours at the overtime rate of pay, and 8 hours at the weekend rate of pay.

b. Complete the following matrix calculation showing the total amount Vince has been paid for this week.

$$\begin{bmatrix} 28 & 6 & 8 \end{bmatrix} \begin{bmatrix} 36 \\ 54 \\ 72 \end{bmatrix}$$

[1908]

$$\begin{bmatrix} 28 & 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} 36. \\ 54. \\ 72. \end{bmatrix} \qquad \begin{bmatrix} 1908. \end{bmatrix}$$

Vince will receive $90 per hour if he works a public holiday shift.

Matrix Q, as calculated below, can be used to show Vince's hourly rate for each type of shift.

$$Q = n \times \begin{bmatrix} 1 & 1.5 & 2 & p \end{bmatrix}$$
$$= \begin{bmatrix} 36 & 54 & 72 & 90 \end{bmatrix}$$

c. Write the values of n and p in the boxes below.

$$\text{solve}\left(\begin{bmatrix} 36 \\ 54 \\ 72 \\ 90 \end{bmatrix} = n * \begin{bmatrix} 1 \\ 1.5 \\ 2 \\ p \end{bmatrix}, \{n, p\}\right)$$

{n=36, p=2.5}

$$\text{solve}\left(\begin{bmatrix} 36 & 54 & 72 & 90 \end{bmatrix} = n \cdot \begin{bmatrix} 1 & 1.5 & 2 & p \end{bmatrix}, n, p\right) \qquad n=36. \text{ and } p=2.5$$

Question 10 (2 marks)

To access the southern end of the construction site, Vince must enter a security code consisting of five numbers.

The security code is represented by the row matrix W.

The element in row i and column j of W is w_{ij}.

The elements of W are determined by the rule $(i-j)^2 + 2j$.

a. Complete the following matrix showing the five numbers in the security code.

$$\text{constructMat}\left((i-j)^2 + 2 \cdot j, i, j, 1, 5\right) \qquad \begin{bmatrix} 2. & 5. & 10. & 17. & 26. \end{bmatrix}$$

createM((i−j)^2+2j, 1, 5)

[2 5 10 17 26]

To access the northern end of the construction site, Vince enters a different security code, consisting of eight numbers.

This security code is represented by the row matrix X.

The element in row i and column j of X is x_{ij}.

The elements of X are also determined by the rule $(i-j)^2 + 2j$.

b. What is the last number in this security code to access the northern end of the construction site?

$$\text{constructMat}\left((i-j)^2 + 2 \cdot j, i, j, 1, 8\right) \qquad \begin{bmatrix} 2. & 5. & 10. & 17. & 26. & 37. & 50. & 65. \end{bmatrix}$$

createM((i−j)^2+2j, 1, 8)

[2 5 10 17 26 37 50 65]

r1c8=(1−8)^2+2×8

r1c8=65

Question 11 (3 marks)

A population of a native animal species lives near the construction site.

This information is displayed in the initial population matrix, R_0, and the Leslie matrix, L, below.

$$R_0 = \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix} \qquad L = \begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$

ii. complete the following table, showing the initial female population, and the predicted female population after one year, for each of the age groups.

$$\begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix}$$

$$\begin{bmatrix} 124 \\ 28 \\ 56 \\ 45 \end{bmatrix}$$

b. It is predicted that if this species is not protected, the female population of each of the four age groups will rapidly decrease within the next 10 years.

After how many years is it predicted that the total female population of this species will first be half the initial female population?

$$\begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix} \times \text{ans}$$

$$\begin{bmatrix} 64.9616 \\ 29.8736 \\ 23.0272 \\ 13.02 \end{bmatrix}$$

Question 12 (4 marks)

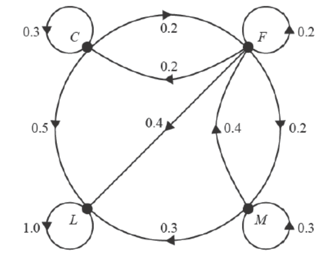
When the construction company established the construction site at the beginning of 2023, it employed 390 staff to work on the site.

This situation can be modelled by the recurrence relation

$$S_{n+1} = TS_n, \text{ where}$$

T is the transitional matrix, $S_0 = \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} \begin{matrix} C \\ F \\ M \\ L \end{matrix}$ and n is the number of years after 2023.

The transition diagram below shows the proportion of staff who are expected to change their job at the site each year.



a. Calculate the predicted percentage decrease in the number of foremen (F) on the site from 2023 to 2025.

$$\begin{bmatrix} 0.3 & 0.2 & 0 & 0 \\ 0.2 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.5 & 0.4 & 0.3 & 1 \end{bmatrix}^2 \cdot \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 48.7 \\ 43. \\ 19.9 \\ 278.4 \end{bmatrix}$$

$$perdec = \frac{50-43}{50} \cdot 100 \quad perdec = 14.$$

b. Determine the total number of staff on the site in the long term.

$$\begin{bmatrix} 0.3 & 0.2 & 0 & 0 \\ 0.2 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.5 & 0.4 & 0.3 & 1 \end{bmatrix}^{40} \cdot \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1.08213051029\text{E-}7 \\ 1.62319576543\text{E-}7 \\ 1.08213051029\text{E-}7 \\ 389.999999621 \end{bmatrix}$$

Matrix R below shows the ways in which staff are expected to change their jobs from year to year with this new incentive in place.

$$R = \overset{\textit{this year}}{\overset{\begin{matrix} C & F & M & L \end{matrix}}{\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1 \end{bmatrix}}} \begin{matrix} C \\ F \\ M \\ L \end{matrix} \quad \textit{next year}$$

The site always requires at least 330 construction workers.

To ensure that this happens, the company hires an additional 190 construction workers (C) at the beginning of 2024 and each year thereafter.

The matrix V_{n+1} will then be given by

$$V_{n+1} = RV_n + Z, \text{ where}$$

$V_0 = \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} \begin{matrix} C \\ F \\ M \\ L \end{matrix}$ $\quad Z = \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} C \\ F \\ M \\ L \end{matrix}$ and n is the number of years after 2023.

c. How many more staff are there on the site in 2024 than there were in 2023?

$$\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} + \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 332. \\ 146. \\ 13. \\ 89. \end{bmatrix}$$

$$332+146+13-(330+50+10) \quad 101.$$

d. Based on this new model, the company has realised that in the long term there will be more than 200 foremen on site.

In which year will the number of foremen first be above 200?

$$\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1 \end{bmatrix} \cdot \text{Ans} + \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 364.24 \\ 187.48 \\ 43.37 \\ 364.91 \end{bmatrix} + \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 373.192 \\ 200.54 \\ 50.507 \\ 525.761 \end{bmatrix}$$

Networks

Question 39

Anush, Blake, Carly and Dexter are workers on a construction site. They are each allocated one task.
The time, in hours, it takes for each worker to complete each task is shown in the table below.

	Task 1	Task 2	Task 3	Task 4
Anush	12	8	16	9
Blake	10	7	15	10
Carly	11	10	18	12
Dexter	10	14	16	11

His completion times, in hours, are listed below.

	Task 1	Task 2	Task 3	Task 4
Edgar	9	5	14	8

When a new allocation is made and Edgar takes over one of the tasks, the minimum total completion time compared to the initial allocation will be reduced by

$$\text{HunAmin}\left(\begin{bmatrix}12 & 8 & 16 & 9\\ 10 & 7 & 15 & 10\\ 11 & 10 & 18 & 12\\ 10 & 14 & 16 & 11\end{bmatrix}\right)$$

43

Assignment matrix =
[[1,4],[2,2],[3,1],[4,3]]
Minimum cost =
43

Transformed Matrix → TransM
Optimal Combination → ResultM